

Name: _____ Date: _____ Period: _____

Cape Florida Lighthouse: Lore and Calculations

Activity 1 - Original Lighthouse

Given: The historic Cape Florida Lighthouse, located in Bill Baggs Cape Florida State Park, was originally built in 1825. After being struck by a hurricane in 1835 and burned by Seminole Indians in 1836, the lighthouse was rebuilt in 1846 to match the original published specifications. Joan Gill Blank states on p. 59 of *Key Biscayne - A History of Miami's Tropical Island and the Cape Florida Lighthouse* that the specifications said, "The tower to be round, built of hard brick, all laid in hydraulic cement, 24 feet diameter at the base, 12 feet diameter at the top, height of 65 feet from the surface of the ground to the top of the deck, walls 4 feet thick at the base, to be regularly graduated to 2 feet at the top. Foundation to be laid as deep as may be necessary to make the fabric secure."

Tasks:

1. Identify a common three-dimensional geometric solid (shape) that best represents (models) the lighthouse tower.
2. Use the description, graph paper, and a ruler to create and label a scale drawing of the lighthouse tower.
3. Ignoring that there are windows and a door, estimate the volume of the bricks and mortar in the lighthouse tower.
4. Suppose the bricks measure $\frac{3}{4}$ " \times $3\frac{3}{8}$ " \times 2" and have $\frac{3}{8}$ " mortar joints; approximate the number of bricks used to construct the lighthouse tower.
5. Determine the lateral surface area, in square feet, of the exterior of the lighthouse tower.
6. Suppose one gallon of paint covers approximately 350 square feet. Calculate the number of gallons of paint needed to cover the exterior of the lighthouse tower with one coat of paint.

Name: _____ **ANSWER KEY** _____ Date: _____ Period: _____

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Tasks:

1. Identify a common three-dimensional geometric solid (shape) that best represents (models) the lighthouse tower. **A truncated cone, a cone with a portion of the pointed part removed, or a conical frustum**
2. Use the description, graph paper, and a ruler to create and label a scale drawing of the lighthouse tower. **See last page**
3. Ignoring that there are windows and a door, estimate the volume of the bricks and mortar in the lighthouse tower.

$$\text{Volume of lighthouse tower} = V_{\text{outer frustum}} - V_{\text{inner frustum}}$$

$$\text{Vol. of outer frustum} = \frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2) \quad \text{Vol. of inner frustum} = \frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$$

$$V = \frac{1}{3}\pi(65)(12^2 + (12)(6) + 6^2)$$

$$V = \frac{1}{3}\pi(65)(8^2 + (8)(4) + 4^2)$$

$$V = \frac{1}{3}\pi(65)(252)$$

$$V = \frac{1}{3}\pi(65)(112)$$

$$V = 5460\pi$$

$$V = 2426.666\pi$$

$$V \approx 17,153.095 \text{ cubic feet}$$

$$V \approx 7,623.598 \text{ cubic feet}$$

Volume of lighthouse tower $\approx 17,153.095 - 7,623.598 \approx \mathbf{9,529.497 \text{ cubic feet}}$

4. Suppose the bricks measure $\frac{3}{4}$ " \times $3\frac{3}{8}$ " \times 2" and have $\frac{3}{8}$ " mortar joints; approximate the number of bricks used to construct the described lighthouse tower.

$$\text{Volume of one brick} = 7\frac{3}{4}" \times 3\frac{3}{8}" \times 2" = 52.3125 \text{ in}^3$$



Volume of one brick with mortar on **three** sides (an increase is length, width, and height)

$$= \left(7\frac{3}{4} + \frac{3}{8}\right) \times \left(3\frac{3}{8} + \frac{3}{8}\right) \times \left(2 + \frac{3}{8}\right) = 72.363 \text{ in}^3$$

Number of bricks = (Volume of lighthouse tower \div Volume of one brick with mortar)

Note: 1 cubic foot = 12 inches \times 12 inches \times 12 inches, so 1 cubic foot = 1,728 cubic inches

$$9529.497 \text{ cubic feet} \times 1728 \text{ cubic inches per foot} = 16,466,970.82 \text{ cubic inches}$$

$$16,466,970.82 \div 72.363 = 227,560.643 \approx \mathbf{227,561 \text{ bricks}}$$

5. Determine the lateral surface area, in square feet, of the exterior of the lighthouse tower.

$$\text{Lateral Surface Area} = F = \pi(r_1 + r_2)\sqrt{h^2 + (r_1 - r_2)^2}$$

$$F = \pi(12 + 6)\sqrt{65^2 + (12 - 6)^2}$$

$$F = \pi(18)\sqrt{4261}$$

$$F \approx \mathbf{3,691.289 \text{ square feet}}$$

6. Suppose one gallon of paint covers approximately 350 square feet. Calculate the number of gallons of paint needed to cover the exterior of the lighthouse tower with one coat of paint.

$$3,691.289 \div 350 = 10.54 \approx \mathbf{11 \text{ gallons}}$$

Profile of Original Cape Florida Lighthouse Tower

